Physics 4A
Chapter 15: Oscillations

"For every minute you are angry, you lose 60 seconds of happiness." – Ralph Waldo Emerson

"Consider how much more often you suffer from your anger and grief than from those very things for which you are angry and grieved." – Marcus Aurelius

"Holding on to anger is like grasping a hot coal with the intent of throwing it at someone else; you are the one who gets burned." – Buddha

Reading: pages 390 – 413

Outline:

⇒ simple harmonic motion (SHM)
    oscillations
    frequency and period
⇒ SHM and circular motion
    the phase constant
⇒ energy in SHM
⇒ the dynamics of SHM
⇒ vertical oscillations
⇒ pendulums
    simple pendulum
    small angle approximation
    physical pendulum
⇒ forced oscillations and resonance

Problem Solving

Some problems make use of the relationships among angular frequency, frequency, and period for simple harmonic motion: $\omega = 2\pi f$, $f = 1/T$, and $\omega = 2\pi/T$. Occasionally the period is given indirectly by describing a time interval. You must then know, for example, that the time the oscillator takes to go from maximum displacement in one direction to maximum displacement in the other direction is $T/2$ or the time it takes to go from maximum displacement to zero displacement is $T/4$. If these time intervals or others are given, you should be able to calculate the period, frequency, and angular frequency. You should also know how to find the maximum speed and maximum acceleration in terms of the angular frequency and amplitude: $v_m = \omega x_m$ and $a_m = \omega^2 x_m$. Some problems require you to know the relationship between the angular frequency and the appropriate physical properties of the oscillating system: $\omega = \sqrt{k/m}$ for an undamped spring-object system.

A problem statement might give you an expression for the displacement of an oscillating object as a function of time and ask for the amplitude, angular frequency, and phase constant (or related quantities). Simply identify the various constants in the given expression. It might also ask for

the coordinate, velocity, and acceleration at some specific time. Simply evaluate the expression and its first and second derivatives for that value of the time.
Some problems can be solved using the principle of mechanical energy conservation. For an spring-object system the mechanical energy E_{mec}, the speed v, and the coordinate x are related by $E_{mec} = ½ mv^2 + ½ kx^2$. If the object has speed v_1 when it is at x_1 and speed v_2 when it is at x_2, then conservation of mechanical energy yields $½ mv_2^2 + ½ kx_2^2 = ½ mv_1^2 + ½ kx_1^2$. Either of these equations can be solved for one of the quantities that appear in them.

Some problems deal with the other oscillating systems discussed in the text: the simple pendulum and the physical pendulum. In each case, you should know how the angular frequency depends on properties of the oscillating body: $\sqrt{g/L}$ for a simple pendulum and $\sqrt{mgh/I}$ for a physical pendulum.

Mathematical Skills

Derivatives of sinusoidal functions.

You need to know how to differentiate $\sin(\omega t + \phi)$ and $\cos(\omega t + \phi)$ to verify the solutions to several of the equations of motion in the chapter and to calculate the velocity and acceleration of an oscillating body. Remember how to use the chain rule. Let $\omega t + \phi = u$. Then, $d\sin(\omega t + \phi)/dt = (d\sin u/du)(du/dt) = (\cos u)(\omega) = \omega \cos(\omega t + f)$. Similarly, $d\cos(\omega t + \phi)/dt = -\omega \sin(\omega t + \phi)$.

Small angle approximation.

Several of the oscillators discussed in this chapter are harmonic only if the amplitude is small. Simple and physical pendulums are examples. For the motion to be considered harmonic the angle θ of swing must be sufficiently small that $\sin\theta$ may be replaced by θ in radians without generating unacceptable error.

The Maclaurin series for $\sin\theta$ is

$$\sin\theta = \sum_{n=0}^{\infty} (-1)^n \frac{\theta^{2n+1}}{(2n+1)!}$$

and its first three terms are

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - + \ldots$$

The series is valid only if θ is in radians. Notice that if θ is small ($\theta \ll 1$), each term in the series is much less in magnitude than the previous term. The small angle approximation amounts to using only the first term of the series.

If θ is small, the error generated by the small angle approximation is nearly the second term $\theta^3/6$ and the fractional error is roughly $\theta^2/6$. For example, the error is less than 1 per cent if $\theta^2/6 < 0.01$ or $\theta < 0.2$ radians. In fact the error for $\theta = 0.2$ radians is 0.7 per cent.

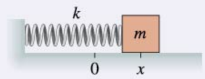
GENERAL PRINCIPLES

Dynamics

SHM occurs when a **linear restoring force** acts to return a system to an equilibrium position.

Horizontal spring

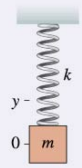
$(F_{\text{net}})_x = -kx$



Vertical spring

The origin is at the equilibrium position $\Delta L = mg/k$.

$(F_{\text{net}})_y = -ky$

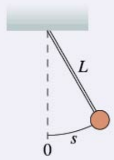


Both: $\omega = \sqrt{\dfrac{k}{m}} \qquad T = 2\pi\sqrt{\dfrac{m}{k}}$

Simple pendulum

$\omega = \sqrt{\dfrac{g}{L}}$

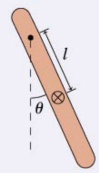
$T = 2\pi\sqrt{\dfrac{L}{g}}$



Physical pendulum

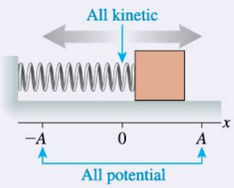
$\omega = \sqrt{\dfrac{Mgl}{I}}$

$T = 2\pi\sqrt{\dfrac{I}{Mgl}}$



Energy

If there is no friction or dissipation, kinetic and potential energy are alternately transformed into each other, but the total mechanical energy $E = K + U$ is conserved.

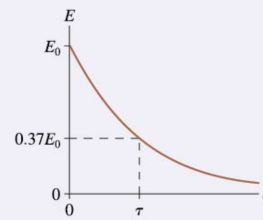


$$E = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2$$
$$= \tfrac{1}{2}m(v_{\max})^2$$
$$= \tfrac{1}{2}kA^2$$

The energy of a lightly damped oscillator decays exponentially

$$E = E_0 e^{-t/\tau}$$

where τ is the **time constant**.



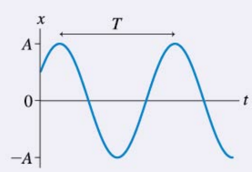
IMPORTANT CONCEPTS

Simple harmonic motion (SHM) is a sinusoidal oscillation with period T and amplitude A.

Frequency $f = \dfrac{1}{T}$

Angular frequency

$$\omega = 2\pi f = \frac{2\pi}{T}$$



Position $x(t) = A\cos(\omega t + \phi_0)$

$$= A\cos\left(\frac{2\pi t}{T} + \phi_0\right)$$

Velocity $v_x(t) = -v_{\max}\sin(\omega t + \phi_0)$ with maximum speed $v_{\max} = \omega A$

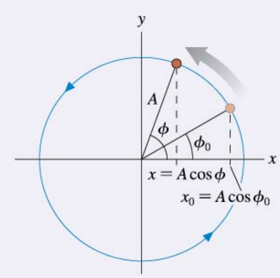
Acceleration $a_x(t) = -\omega^2 x(t) = -\omega^2 A\cos(\omega t + \phi_0)$

SHM is the projection onto the x-axis of **uniform circular motion.**

$\phi = \omega t + \phi_0$ is the **phase**

The position at time t is

$$x(t) = A\cos\phi$$
$$= A\cos(\omega t + \phi_0)$$



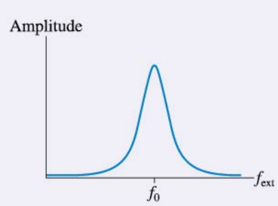
The **phase constant** ϕ_0 is determined by the initial conditions:

$$x_0 = A\cos\phi_0 \qquad v_{0x} = -\omega A\sin\phi_0$$

APPLICATIONS

Resonance

When a system is driven by a periodic external force, it responds with a large-amplitude oscillation if $f_{\text{ext}} \approx f_0$, where f_0 is the system's natural oscillation frequency, or **resonant frequency.**

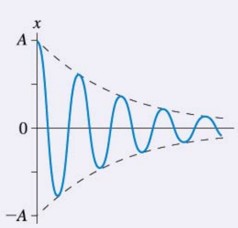


Damping

If there is a drag force $\vec{F}_{\text{drag}} = -b\vec{v}$, where b is the damping constant, then (for lightly damped systems)

$$x(t) = Ae^{-bt/2m}\cos(\omega t + \phi_0)$$

The time constant for energy loss is $\tau = m/b$.



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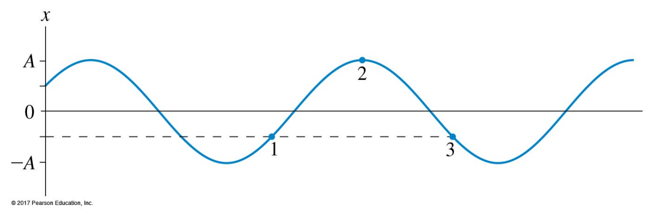
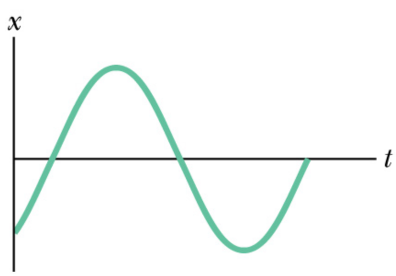
Questions and Example Problems from Chapter 15

Conceptual Question 15.2
A pendulum on Planet X, where the value of g is unknown, oscillates with a period T = 2 s. What is the period of this pendulum if: **(a)** Its mass is doubled? **(b)** Its length is doubled? **(c)** Its oscillation amplitude is doubled?

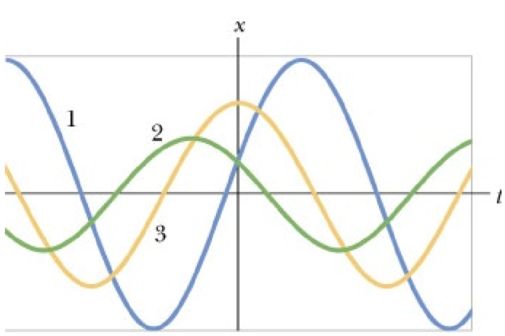
Conceptual Question 15.4
The figure shows a position-versus-time graph for a particle in SHM. **(a)** What is the phase constant? *Explain.* **(b)** What is the phase of the particle at each of the three numbered points on the graph?



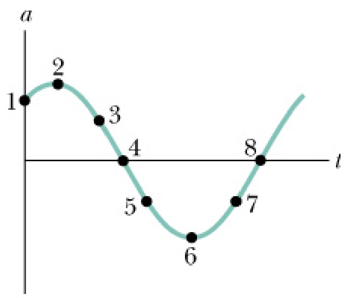
Conceptual Question 15.A
Which of the following describe ϕ for the SHM of the figure below **(a)** $-\pi < \phi < -\pi/2$,
(b) $\pi < \phi < 3\pi/2$, **(c)** $-3\pi/2 < \phi < -\pi$?



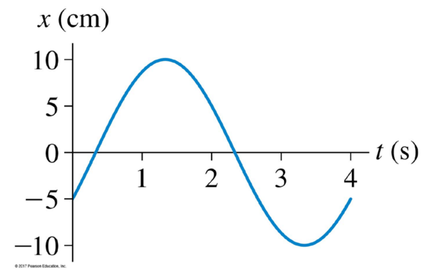
Conceptual Question 15.B
The figure below shows the x(t) curves for three experiments involving a particular spring-mass system oscillating in SHM. Rank the curves according to **(a)** the system's angular frequency; **(b)** the spring's potential energy at time t = 0, **(c)** the box's kinetic energy at t = 0, **(d)** the masses speed at t = 0, and **(e)** the masses maximum kinetic energy, greatest first.



Conceptual Question 15.C
The acceleration a(t) of a particle undergoing SHM is graphed in the figure below. **(a)** Which of the labeled points corresponds to the particle at $-x_m$? **(b)** At point 4, is the velocity of the particle positive, negative, or zero? **(c)** At point 5, is the particle at $-x_m$, at $+x_m$, at 0, between $-x_m$ and 0, or between 0 and $+x_m$?



Problem 15.6
What are the **(a)** amplitude, **(b)** frequency, and **(c)** phase constant of the oscillation shown in the figure to the right?



Problem 15.9
An object in simple harmonic motion has an amplitude of 4.0 cm, a frequency of 2.0 Hz, and a phase constant of $2\pi/3$ rad. Draw a position graph showing two cycles of motion.

Problem 15.11
An object in simple harmonic motion has amplitude 4.0 cm and frequency 4.0 Hz, and at t = 0 s it passes through the equilibrium point moving to the right. Write the function x(t) that describes the object's position.

Problem 15.16
A 200 g mass attached to a horizontal spring oscillates at a frequency of 2.0 Hz. At t = 0 s, the mass is at x = 5.0 cm and has v_x = -30 cm/s. Determine: **(a)** The period. **(b)** The angular frequency. **(c)** The amplitude. **(d)** The phase constant. **(e)** The maximum speed. **(f)** The maximum acceleration. **(g)** The total energy. **(h)** The position at t = 0.40 s.

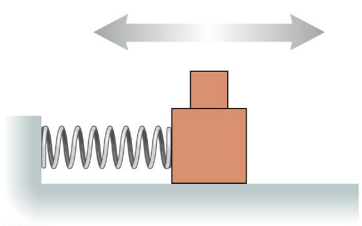
Problem 15.18
A 1.0 kg block is attached to a spring with spring constant 16 N/m. While the block is sitting at rest, a student hits it with a hammer and almost instantaneously gives it a speed of 40 cm/s. What are **(a)** The amplitude of the subsequent oscillations? **(b)** The block's speed at the point where x = ½ A.

Problem 15.26
A mass on a string of unknown length oscillates as a pendulum with a period of 4.0 s. What is the period if: **(a)** The mass is doubled? **(b)** The string length is doubled? **(c)** The string length is halved? **(d)** The amplitude is doubled? (Parts a to d are independent questions, each referring to the initial situation.)

Problem 15.29
Astronauts on the first trip to Mars take along a pendulum that has a period on earth of 1.50 s. The period on Mars turns out to be 2.45 s. What is the free-fall acceleration on Mars?

Problem 15.46
A 200 g block hangs from a spring with spring constant 10 N/m. At t = 0 s the block is 20 cm below the equilibrium point and moving upward with a speed of 100 cm/s. What are the block's **(a)** Oscillation frequency? **(b)** Distance from equilibrium when the speed is 50 cm/s? **(c)** Distance from equilibrium at t = 1.0 s?

Problem 15.52
The two blocks in the figure oscillate on a frictionless surface with a period of 1.5 s. The upper block just begins to slip when the amplitude is increased to 40 cm. What is the coefficient of static friction between the two blocks?



Problem 15.60
A 500 g air-track glider attached to a spring with spring constant 10 N/m is sitting at rest on a frictionless air track. A 250 g glider is pushed toward it from the far end of the track at a speed of 120 cm/s. It collides with and sticks to the 500 g glider. What are the amplitude and period of the subsequent oscillations?

Problem 15.A
An oscillator consists of a block of mass 0.500 kg connected to a spring. When set into oscillation with amplitude 35.0 cm, the oscillator repeats its motion every 0.500 s. Find (a) the period, (b) the frequency, (c) the angular frequency, (d) the spring constant, (e) the maximum speed, and (f) the magnitude of the maximum force on the block from the spring.

Problem 15.B
A body oscillates with simple harmonic motion according to the equation:
$x = (6.0\,\text{m})\cos[(3\pi\ \text{rad/s})t + \pi/3\,\text{rad}]$. At t = 2.0 s, what are (a) the displacement, (b) the velocity, (c) the acceleration, and (d) the phase of the motion? Also, what are (e) the frequency and (f) the period of the motion?

Problem 15.C
A simple harmonic oscillator consists of a block of mass 2.00 kg attached to a spring of spring constant 100 N/m. When t = 1.00 s, the position and velocity of the block are x = 0.129 m and v = 3.415 m/s. (a) What is the amplitude of the oscillations? What were the (b) position and (c) velocity of the block at t = 0 s?